How to Do Proofs in Mathematics?

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1 Notes on Proving Statements

As economists, many results we need to prove are straightforward, in the sense that we only need to apply definitions again and again. Keep in mind that theorems are essentially restatements of definitions. Therefore what is critical to successful proofs is a good understanding of related definitions, as well as some experience.

Although the best way to learn proving statements is by doing it, there are a few guidelines for beginners.

1. Rewrite all notations and terminologies in the statement we want to prove in terms of their definitions.

2. At the beginning of a complicated proof, write down what we want to show to avoid getting lost later on.

3. Proving a "for any" statement

   When proving that some statement related to x is correct for any $x \in X$, start the proof by writing down "Take any $x \in X$", and then try to get to statement we want to prove through a flow of logic that works for all $x \in X$.

4. Existence proofs

   When proving that there exists at least one $x \in X$ that satisfies some properties, we often do it by construction (although there are also non-constructive existence proofs, which is usually more advanced). That is, we explicitly construct a specific x, and then try to show that x we have constructed satisfies those required properties.

   Existence proofs tend to be more difficult, since the required constructions may not be so straightforward. Sometimes trial and error are necessary when coming up with a construction that really works.

5. Uniqueness proofs

   When proving that there exists at most one $x \in X$ that satisfies some properties, we start by taking any x_1 and x_2 that both satisfy those properties, and then try to get to the conclusion that they must be the same element.

6. Proofs by contradiction

   These proofs are done by hypothesizing that what we want to show is not true, and then try to get contradiction, which allows us to conclude that the hypothesis we started with must be wrong.

   This is a useful technique, because sometimes it is hard to start the proof directly, possibly because the conditions we have are not easy to start with. By hypothesizing that what we want to show is not true, we get one more starting point of our reasoning.

7. Invoke known results

   The results that we know are true, possibly because we have proved them, can be directly used to prove more complicated results. It is not necessary to prove every single statement using the first principles.

   Learning more proof techniques is of course important. But what is more important for people like us who just want to apply mathematics to solve real life problems is to accumulate a full supply of useful known results and be ready to apply them.